

→ 3. highest quizzes will be taken.

finding a basis for a subspace

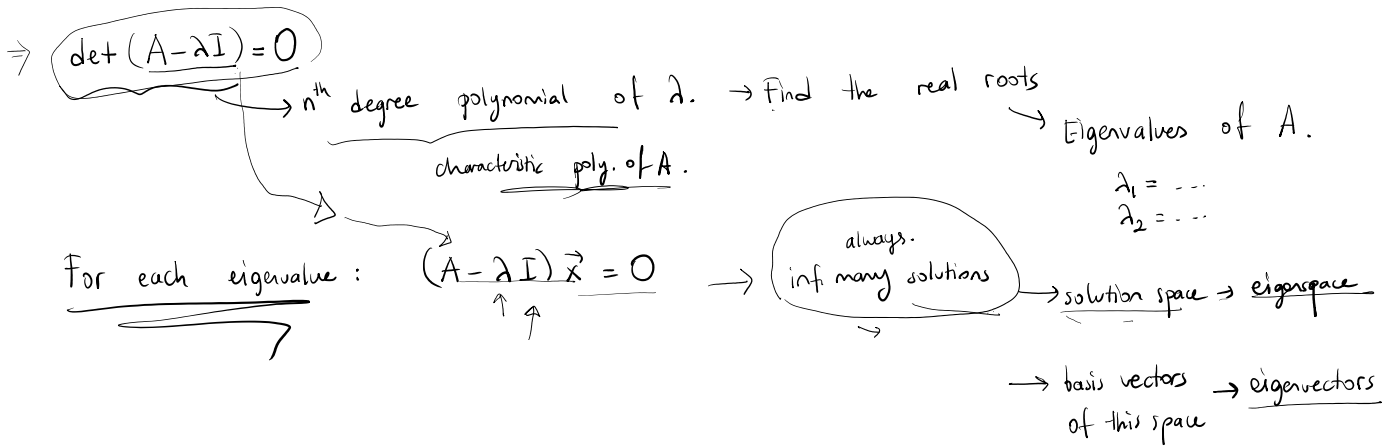
Diagonalization

$A_{n \times n}$

eigenvalues

eigenspaces

eigenvectors



Diagonalization:

$$A = X D X^{-1}$$

$\begin{matrix} \swarrow & \searrow \\ \text{an invertible} & \text{a diagonal matrix} \\ \text{matrix} & \end{matrix}$

$$\begin{bmatrix} d_1 & & \\ & d_2 & \\ & & \ddots \\ & & & d_n \end{bmatrix}$$

⇒ If we are able to write A in this form \Downarrow
 A is "diagonalizable"!

We'll make use of eigenvalues and eigenvectors.

1) If $A_{n \times n}$ has n distinct eigenvalues $\Rightarrow A$ is diagonalizable!

$\lambda_1, \lambda_2, \dots, \lambda_n$

$$D = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{bmatrix}_{n \times n}$$

$$X = \begin{bmatrix} | & | & & | \\ | & | & \dots & | \\ | & | & & | \\ | & | & & | \end{bmatrix}_{n \times n}$$

↑
 the corresponding eigenvectors for λ_1, λ_2 in the same order with D .

$$\boxed{A = X D X^{-1}}$$

2) If $A_{n \times n}$ has less than n distinct eigenvalues $\Rightarrow A$ may or may not be diagonalizable

2) If $A_{n \times n}$ has less than n distinct eigenvalues \Rightarrow A may or may not be diagonalizable
(we have multiple roots for λ 's) \Rightarrow We should find all eigenvectors

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \dots \end{bmatrix} \quad X = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$\lambda_3 \rightarrow$ multiplicity 2

\Rightarrow if we have n lin. ind. eigenvectors in total
 \Rightarrow A is diagonalizable.
Otherwise, A is NOT diagonalizable.

Ex