14th Week Wednesday

24 Mayıs 2023 Çarşamba 14:32

1st Section → 6-7 questions — 2nd Midterm — 45 min
5 min break →
2nd Section → 8-9 questions — Final Exam — 60 min

105-110 min

finding a basis for a subspace

→ 3 highest quizzes will be taken.

Diagonalization

$A_{n \times n}$

eigenvalues eigenspaces eigenvectors

⇒ $\det(A - \lambda I) = 0$ → n^{th} degree polynomial of λ. → Find the real roots → Eigenvalues of A.

characteristic poly. of A.

$\lambda_1 = \dots$
$\lambda_2 = \dots$

For each eigenvalue : $(A - \lambda I)\vec{x} = 0$ → always. inf many solutions → solution space → eigenspace

→ basis vectors of this space → eigenvectors

Diagonalization : $A_{n\times n} = X_{n\times n} D_{n\times n} X^{-1}_{n\times n}$ ⇒ If we are able to write A in this form ⇒ A is "diagonalizable"!

X: an invertible matrix

D: a diagonal matrix $\begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}$

We'll make use of eigenvalues and eigenvectors.

1) If $A_{n\times n}$ has n distinct eigenvalues ⇒ A is diagonalizable!

$\lambda_1, \lambda_2, \dots, \lambda_n$ → $D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}_{n\times n}$ $X = \begin{bmatrix} | & | & \dots & | \\ | & | & & | \end{bmatrix}_{n\times n}$

the corresponding eigenvectors for λ_1, λ_2 in the same order with D.

$$\boxed{A = XDX^{-1}}$$

2) If $A_{n\times n}$ has less than n distinct eigenvalues ⇒ A may or may not be diagonalizable

2) If $A_{n\times n}$ has less than n distinct eigenvalues $\Rightarrow$ A may or may not be diagonalizable
(we have multiple roots for λ's)

$\Rightarrow$ We should find all eigenvectors

$\Rightarrow$ if we have n lin. ind. eigenvectors in total

$\Rightarrow$ A is diagonalizable.

Otherwise, A is NOT diagonalizable.

$$D = \begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \lambda_3 & & \\ & & & \lambda_3 & \\ & & & & \ddots \end{bmatrix} \qquad X = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$\lambda_3 \rightarrow$ multiplicity 2

Ex